Argentina business and technology review
ISSN 3072-7766
2026 · Volumen II
Artículo

1. Mixed-Integer Linear Model for Optimal Class and Classroom Scheduling

Modelo lineal entero-mixto para la programación óptima de clases y aulas

Modelo linear inteiro-misto para o agendamento ótimo de turmas e salas de aula

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Recibido: 4 de enero de 2026
Aceptado: 2 de febrero de 2026

DOI: https://doi.org/10.56487/9emjsv45

Abstract

The task of class scheduling in university settings demands significant organizational effort due to its combinatorial nature. This complexity has been amplified with the adoption of various teaching modalities as online programs gain traction.
While a wide range of academic works aimed at solving this type of problem, commonly known as the University Course Timetabling Problem (UCTP), can be found in the literature, there are few references to any that simultaneously consider the different class delivery modalities currently in use. Moreover, among the published works to solve UCTP problems, models based on Mixed-Integer Linear Programming (MILP) stand out due to their versatility and adaptability to different situations. However, solving these models is computationally intensive.

In the present work, a MILP model adapted to class scheduling under the curriculum planning approach is developed. By considering the student body in an aggregate manner, the model enhances computational tractability, allowing for the simultaneous planning of face-to-face and synchronous virtual classes. Concurrently, a hybrid algorithm is implemented, achieving at least a 100x speedup in computational efficiency and ensuring the model's viability within suitable timeframes for practical use.

Keywords

UCTP — Planning — Optimization

Resumen

La tarea de programar clases en entornos universitarios exige un esfuerzo organizativo debido a su naturaleza combinatoria. Esta complejidad se ha visto amplificada con la adopción de diversas modalidades de enseñanza, a medida que los programas en línea ganan popularidad.
Aunque en la literatura se pueden encontrar numerosos trabajos académicos dirigidos a resolver este tipo de problema, comúnmente conocido como el problema de programación horaria de cursos universitarios (UCTP, por sus siglas en inglés), existen pocas referencias a aquellos que consideren simultáneamente las distintas modalidades de impartición de clases actualmente en uso. Además, entre los trabajos publicados para resolver problemas UCTP, destacan los modelos basados en programación lineal entera mixta (MILP) debido a su versatilidad y capacidad de adaptación a diferentes situaciones. No obstante, resolver estos modelos requiere un alto poder computacional.
En el presente trabajo se desarrolla un modelo MILP adaptado a la programación de clases bajo un enfoque de planificación curricular. Al considerar el alumnado de manera agregada, el modelo mejora la tratabilidad computacional, permitiendo la planificación simultánea de clases presenciales y virtuales sincrónicas. Paralelamente, se implementa un algoritmo híbrido, logrando al menos una aceleración computaciones de 100 veces y asegurando la viabilidad del modelo dentro de plazos adecuados para su uso práctico.

Palabras claves

UCTP — Planificación — Optimización

Resumo

A tarefa de agendar aulas em ambientes universitários exige um esforço organizacional significativo devido à sua natureza combinatória. Essa complexidade foi ampliada com a adoção de diversas modalidades de ensino, à medida que os programas online ganham popularidade.
Embora seja possível encontrar na literatura uma ampla gama de trabalhos acadêmicos voltados à resolução desse tipo de problema, comumente conhecido como Problema de Programação Horária de Cursos Universitários (UCTP, na sigla em inglês), existem poucas referências àqueles que consideram simultaneamente as diferentes modalidades de oferta de aulas atualmente em uso. Além disso, entre os trabalhos publicados para resolver problemas UCTP, destacam-se os modelos baseados em Programação Linear Inteira Mista (MILP) devido à sua versatilidade e capacidade de adaptação a diferentes situações. No entanto, resolver esses modelos é computacionalmente intenso.
No presente trabalho, desenvolve-se um modelo MILP adaptado ao agendamento de aulas sob a abordagem de planejamento curricular. Ao considerar o corpo discente de forma agregada, o modelo melhora a tratabilidade computacional, permitindo o planejamento simultâneo de aulas presenciais e virtuais síncronas. Paralelamente,

implementa-se um algoritmo híbrido, alcançando pelo menos uma aceleração computacional de 100 vezes e garantindo a viabilidade do modelo dentro de prazos adequados para uso prático.

Palavras-chave

UCTP — Planejamento — Otimização

Introduction

The modalities in higher education teaching, where complete in-person attendance prevailed some years ago, have taken unexpected turns after the sanitary conditions imposed by SARS-CoV2. In the university setting, there has been a shift from almost total virtuality during the pandemic period to a curriculum that can have in-person or virtual courses in different proportions depending on the institution's conditions and its target audience. For instance, according to data from the U.S. National Center for Education Statistics, the percentage of students enrolled in at least one course under virtual modality stabilized at around 51% by 2023, while prior to the pandemic this value was around 33% (NCES, 2023). Within distance learning courses, two main types of activities are further differentiated: synchronous and asynchronous. Although both types require a commitment of teaching load from the faculty, it is the former that imposes stronger restrictions on organization, as they require the instructor to coincide in the temporal space with the students. While the latter, in general, require a greater investment prior to the course in preparation of activities, but with fewer restrictions regarding planning activities. It is for this reason that it is important to incorporate the restrictions imposed by synchronous virtual classes into any systematized process of scheduling and classroom planning.

On the other hand, the activity of class scheduling is particularly challenging, because the availability of classrooms and laboratories, faculty time availability, course composition, and several other constraints must be simultaneously considered, which will determine the time and place of each of the classes of the different subjects corresponding to each of the degree programs. Historically, universities have relied on an experienced human team to solve this problem manually on each relevant occasion, which consumes huge amounts of time from the expert team. Due to this, considerable efforts have been dedicated to automating this process.

The problem in question is classified within the so-called planning and/or sequencing problems (Babaei et al., 2015) and, more specifically, as the University Timetabling Problem as it is commonly known in academic circles. Following the celebration of the II International Timetabling Competition in 2007 (ITC2007), it was named the University Course Time Tabling Problem (UCTP). According to Wren (1996), the following definition can be given: University course timetabling involves allocating classes across time and space, respecting human and material constraints, while maximizing compliance with predefined objetives.

Since ITC2007, the UCTP has been differentiated into two possible categories of problems: Post Enrollment-based Course Timetabling and Curriculum-based Course Timetabling (Borchani et al., 2017). In its original formulation, the Post-enrollment problem precisely details conflicts in the schedule of each student, who may choose from a wide variety of courses, and secondly, the suitability of the classrooms required, while overlooking potential conflicts in the teaching staff's schedule (Lewis et al., 2007). To address this gap, the International Timetabling Competition 2019 introduced a feature called *SameAttendees*, which implicitly includes faculty availability, though still with limitations (Müller et al., 2018). Prior to this, Mendez-Díaz et al. (2016) proposed a Post-enrollment model with explicit faculty inclusion; however, running such a model for real-world instances usually required ten hours—largely because each student is considered individually, including academic performance data. In the context of Argentine universities, the Curriculum-based approach is more common, as it considers students in an aggregate manner, making the problem computationally more tractable. Given these characteristics of the Argentine university system, the approach based on the curricular structure of the courses is adopted in this work. It consists of scheduling the corresponding courses to compile a weekly agenda, where each course has subjects taught by departments with one or more faculty members. An adequate space for the department activities and the periods of time when the classes will take place must be assigned to each degree-course-subject-faculty tuple.

UCTP is a complex problem, which is why a variety of approaches have been explored to achieve its solution. Some of the main approaches have been based on metaheuristics, such as: Tabu search (Di Gaspero & Schaerf, 2001; Petrovic & Bykov, 2003); Tabu search in conjunction with Simulated Annealing (Burke et al., 2004; Thompson & Dowsland, 1996); Hill Climbing (Qu et al., 2009); Constraint Programming (Brailsford et al., 1999); VNS Algorithm (Burke et al., 2010; Meyers & Orlin, 2006); Genetic Algorithms (Corne et al., 1994); Ant Colony (Dorigo & Blum, 2005; Qu et al., 2009). On the other hand, there is also an important set of works where Mixed Integer Linear Models are formulated for UCTP. This type of model is widely used because it has the virtue of being relatively easy to modify and extend to consider new situations (Arratia-Martinez et al., 2021; Lemos et al., 2019, 2020; Oladejo et al., 2019; Phillips et al., 2015).

The published MILP models for UCTP are normally centered on the assignment of the time at which faculty and students must synchronously meet for each subject to be taught. It is relatively common that in the development of these models, classrooms are considered to have sufficient capacity, so in principle, they do not constitute a limitation to consider, such as in the models presented in Arratia-Martinez et al. (2021), Hmer & Mouhoub (2010), and Mirhassani (2006).

Since the capacity of each classroom can allow or prevent a certain subject from being taught in a certain classroom, some researchers have proposed models where this restriction is included through the use of sets that link the classrooms that can be used to teach certain subjects and to certain groups of students, such as the model proposed in Daskalaki et al. (2004) and Daskalaki & Birbas (2005). The use of these sets allows for a general representation of the possibility of assigning a certain subject to a determined classroom, including in the same parameter the effect of other requirements, such as the type of classroom (conventional classrooms, laboratories, amphitheaters, etc.) or the need for specific equipment. However, the construction of these sets based on the criteria defined in each educational institution can be a non-trivial problem. In many cases, where the set of classrooms to be assigned has relatively homogeneous characteristics, the possibility of assigning one of them to a certain subject can be carried out by considering only their capacity and the number of students who must be present according to the subject in question. Bearing this in mind, models have been developed where the capacity of the classrooms is explicitly considered as a limiting factor, such as the one presented in (Prabodanie, 2017).

Although, in many cases the teaching modality is defined by external conditions, in other cases it can be a decision variable subject to the availability of classroom spaces. An extreme case of the latter is what occurred due to the capacity restrictions during the SARS-CoV2 pandemic. To date, there are few works that, in addition to classroom capacity, contemplate the teaching modality of the courses as a decision variable. Barnhart presents a method based on mathematical optimization models for class planning that incorporates in-person and synchronous virtual modalities (Barnhart et al., 2022). Using heuristic rules to restrict the search space, they achieved solutions for large-sized instances (620 courses, 4,287 students) in 12 hours. On the other hand, Davison's work presents a hierarchical optimization method also based on mathematical models that incorporate a new modality, called "hybrid", in which one group of students can access classes in person, and another group can do so in a synchronous virtual manner (Davison et al., 2025). This work does not indicate computation times to make a comparison of the method's performance; however, the conclusions of this work seem to indicate that computational complexity is a problem to be solved when adding this new modality. It should be highlighted that the last two works mentioned are designed for Post-enrollment planning approaches, and models designed for planning under the curriculum-based approach are not yet found.

Problem

The most general problem consists then of selecting the teaching modality of each subject *mt*, taught by the faculty member *d*, respecting the established capacities for the classrooms. Subject in turn to a set of restrictions that prevent assigning simultaneous classes to the same faculty member or group of students or in the same physical classroom. This general problem also includes other usual UCTP restrictions, such as faculty availability, or the institution's preferences regarding schedules.

In general, the problem must also maximize some variable of interest to select the most desirable alternatives, this is the *objective function*. Since personal interaction is an important part of the training of future professionals, an objective function can be formulated that rewards solutions where the total number of students receiving in-person classes X is greater, to which other variables can also be added that allow representing the preferences of each institution.

The modalities contemplated in this work are two: the in-person modality, where all students attend a physical classroom, and the synchronous virtual modality, in which all students participate in the class in a synchronous virtual manner. This latter modality is often abbreviated as "online".

Objective

The present work addresses the development of a MILP model under the Curriculum-based approach, leveraging student aggregation to enhance computational tractability and optimize processing times. This efficiency allows the model to handle the simultaneous planning of face-to-face and synchronous virtual modalities while maintaining an adequate distribution of time modules, providing a robust and practical tool for the Argentine university context.

Methods

Due to the high number of combinations among the different decisions normally involved in class scheduling (days, times, classrooms, student groups, and lecture halls), coupled with the requirement to select the teaching modality, the use of computational tools to support the planner becomes indispensable.

This work presents a MILP model that allows finding solutions to the problem posed above, considering the complete set of constraints simultaneously.

The development of the model is carried out by first considering the discrete decisions that must be made, which are represented by binary variables. These variables are linked through algebraic constraints that capture the logical conditions and the hierarchical structure of the discrete decisions. Then, the binary variables are connected to the real variables that represent the continuous decisions of the problem, using Big-M type constraints.

Mathematical Model

The formal description of MILP models is commonly performed using mathematical expressions, which make use of two basic elements: variables and index sets. These, in turn, are used for the formulation of the Objective Function and the multiple Constraints that define the search space.

Sets

The following sets are used for the construction of the mathematical model.

1. *a* = Set of classrooms (includes physical classrooms and the virtual classroom).
2. *mt* = Set of subjects.
3. *dc* = Set of faculty members (teachers).
4. *d* = Set of days.
5. *mh* = Set of time slots (modules).
6. *cr* = Set of academic programs (careers).
7. *c* = Set of student cohorts.
8. *h* = Set of the number of modules (hours) that can be taught per day.
9. *F* = Subset of classrooms, excluding the virtual classroom.
10. *DMH* = Set of tuples that indicate the valid combinations between days and time slots.
11. *DM* = Set of tuples that indicate the valid combinations between faculty members and subjects.
12. *MCC* = Set of tuples that link each subject with the academic programs and cohorts where it is taught.
13. *DDM* = Set of tuples that establish the preferred days and time slots for a given faculty member to teach classes.

Variables

The developed model uses the following variables.

Ydc,mt = Binary variable that is activated when it is decided that a subject *mt* will be taken in the *in-person* modality, while it is deactivated when it will be taught *virtually*.

UPdc,mt,d,a = Variable that must be activated to select the day and classroom in cases where a subject is taught in person.

Wdc,mt,d,mh,a = Binary variable that is activated when choosing to teach a class of the subject *mt*, by faculty member *dc* on day *d*, time slot *mh*, and classroom *a*.

Hdc,mt,d,mh,h = Binary variable that is activated when faculty member *dc* must teach, on day *d*, the subject *mt* for *h* consecutive hours, counted from the beginning of time slot *mh*.

Tcr,c,dc,mt,d,mh = Binary variable that is activated when faculty member *dc* teaches the subject *mt*, to cohort *c* of academic program *cr*, in time slot *mh* on day *d*.

Xdc,mt = Non-negative continuous variable that indicates the number of students who take a subject with a certain faculty member in person.

XOdc,mt = Non-negative continuous variable that indicates the number of students who take a subject with a certain faculty member virtually.

Objective Function

The main objective is to maximize the number of students receiving in-person classes. However, given the combinatorial nature of the problem, many alternative solutions exist with an identical level of in-person attendance. Therefore, it is necessary to include priorities and penalties to guide the search towards those solutions that are more desirable from the educational institution's perspective. Furthermore, the use of these priorities and penalties on the solutions positively affects the convergence speed of the MILP solving algorithms, as it tends to concentrate the search in a more reduced solution space.

A typical case of penalty occurs when seeking to discourage the assignment of classes to teachers outside their desired schedule; for this purpose, we define the auxiliary variable *penalties*, which is introduced as a negative term in the objective function. On the other hand, a case of priority could be considered the institution's policy that seeks to have the highest possible number of hours for each class on a specific day, to minimize daily subject changes for students. For this purpose, the variable *priorities* is introduced, which will be introduced as a positive term in the objective function. Subsequently, the calculation formula for the objective function is expressed in the form of equation (1).

$$FObj = \sum_{(dc,mt)\in DM} X_{dc,mt} + Priorities - Penalties \tag{1}$$

Where priorities and penalties are calculated according to equations (2) and (3).

$$Priorities = \sum_{(dc,mt)\in DM, d, mh, h} PR \cdot h^2 \cdot H_{dc,mt,d,mh,h} \tag{2}$$

$$Penalties = \sum_{a,(dc,mt)\in DM,(d,mh)\in DMH,\forall(dc,d,mh)\notin DDM} PN \cdot W_{dc,mt,d,mh,a} \tag{3}$$

Where *DDM* is the set of combinations of days and modules in which each teacher prefers to teach their courses, while *PR* and *PN* are the weighting factors for the priorities and penalties, respectively.

The objective function is highly adaptable, allowing for the inclusion of institutional priorities or penalties. An example of this is the penalization of class time-windows to achieve higher schedule density, which consequently lowers energy costs associated with HVAC (heating, ventilation, and air conditioning) and lighting.

Constraints

The mathematical model also consists of a set of constraints, which link the different decision variables. These constraints are briefly described in the following paragraphs.

The selection of class modalities for each subject *mt*, taught by the teacher *dc*, is carried out by modifying the value of the variable *Ydc,mt*. A value of 1 for this variable indicates a face-to-face class modality, while a value of 0 indicates that the class will be taught in synchronous virtual modality.

For each teacher-subject-day combination, a maximum of one classroom can be selected, which is established by equation (4), which controls the selection of classrooms for face-to-face classes. This is complemented by equation (5), which prevents the selection of virtual classrooms for this modality.

$$\sum_{a\neq Virtual} UP_{dc,mt,d,a} \leq 1 \quad \forall(dc,mt)\in DM, d \tag{4}$$

$$UP_{dc,mt,d,a} = 0 \quad \forall(dc,mt)\in DM, d, a = Virtual \tag{5}$$

A basic constraint of the UCTP is to ensure that each teacher is assigned the necessary number of time slots (modules) to cover the teaching load for each subject they are responsible for. For this purpose, in the case of the proposed model, constraints (6) and (7) are introduced.

$$\sum_{d,mh,a\mid(dc,mt)\in DM,(d,mh)\in DMH} W_{dc,mt,d,mh,a} = DMC_{dc,mt} \quad \forall(dc,mt)\in DM \tag{6}$$

$$\sum_{d,mh\mid(d,mh)\in DMH} T_{cr,c,dc,mt,d,mh} = DMC_{dc,mt} \quad \forall dc,mt,cr,c\mid(dc,mt)\in DM,(mt,cr,c)\in MCC \tag{7}$$

Equation (8) specifies that at each given moment determined by the day-module tuple *(d,mh)*, there can be at most one lecturer-subject tuple *(dc,mt)* assigned to each student group, expressed by the career-cohort pair *(cr,c)*.

$$\sum_{(dc,mt)\in DM:(mt,cr,c)\in MCC} T_{cr,c,dc,mt,d,mh} \leq 1 \quad \forall(d,mh)\in DMH,(cr,c) \tag{8}$$

The constraint defined in equation (9) indicates to the model that at any given time defined by the tuple *(d,mh)* and for any classroom *(a)*, a single teacher *(dc)* can teach only one subject *(mt)*.

$$\sum_{a,\,mt\,|\,(dc,\,mt)\in DM} W_{dc,mt,d,mh,a} \leq 1 \quad \forall (d,mh) \in DMH, dc \tag{9}$$

Similarly, in each physical classroom and at any time *(d,mh)*, at most one class can be taught, which is expressed by restriction (10).

$$\sum_{dc,\,mt\,|\,(dc,\,mt)\in DM} W_{dc,mt,d,mh,a} \leq 1 \quad \forall (d,mh) \in DMH, a \neq Virtual \tag{10}$$

When a specific course is selected to be taught in person, the number of students who will receive it online is zero, which is expressed in equation (11). Furthermore, the number of students who must take said course in person will be equal to the total number of enrolled students. This condition is expressed by the pair of constraints (12) and (13).

$$XO_{dc,mt} \leq +M_1 \cdot (1 - Y_{dc,mt}) \quad \forall (dc,mt) \in DM \tag{11}$$

$$X_{dc,mt} - MAI_{mt} \leq +M_1 \cdot (1 - Y_{dc,mt}) \quad \forall (dc,mt) \in DM \tag{12}$$

$$X_{dc,mt} - MAI_{mt} \geq -M_1 \cdot (1 - Y_{dc,mt}) \quad \forall (dc,mt) \in DM \tag{13}$$

When a specific subject is selected to be taught online, the number of students receiving it face-to-face is zero; this is expressed in equation (14). In turn, the number of students who must attend said subject face-to-face will be equal to the total number of enrolled students. This condition is expressed by the pair of constraints (15) and (16).

$$X_{dc,mt} \leq +M_1 \cdot Y_{dc,mt} \quad \forall (dc,mt) \in DM \tag{14}$$

$$XO_{dc,mt} - MAI_{mt} \leq +M_1 \cdot Y_{dc,mt} \quad \forall (dc,mt) \in DM \tag{15}$$

$$XO_{dc,mt} - MAI_{mt} \geq -M_1 \cdot Y_{dc,mt}, \quad \forall (dc,mt) \in DM \tag{16}$$

Furthermore, under the in-person modality, the number of students enrolled in the course must not exceed the capacity of the assigned classroom. However, since only in-person students are admitted, the previous condition can be represented by constraint (17).

$$X_{dc,mt} - AC_a \leq +M_1 \cdot (1 - UP_{dc,mt,d,a}) \quad \forall (dc,mt) \in DM, d, a \neq Virtual \tag{17}$$

In the case of subjects with a relatively high number of weekly hours, it is common to prefer distributing the workload across two or more days, thereby limiting the hours of said subjects to no more than *N* hours per day. This condition can be expressed by restrictions (18) and (19), which apply to face-to-face and virtual classes, respectively.

$$\sum_{mh} W_{dc,mt,d,mh,a} - N \leq +M_2 \cdot (1 - Y_{dc,mt,'P'}) \quad \forall (dc,mt) \in DM, d, a \neq Virtual \tag{18}$$

$$\sum_{mh} W_{dc,mt,d,mh,a} - N \leq +M_2 \cdot Y_{dc,mt,'P'} \quad \forall (dc,mt) \in DM, d, a = Virtual \tag{19}$$

When a subject is designated to be taught in virtual mode, and assuming that the instructor has an adequate space to carry out the transmission without requiring a classroom, the combinations of classes to be taught in physical classrooms must then be excluded from the search space. This condition is expressed by constraint (20).

$$Y_{dc,mt,'P'} - W_{dc,mt,d,mh,a} \geq 0 \quad \forall (dc,mt) \in DM, (d,mh) \in DMH, a \neq Virtual \tag{20}$$

On the other hand, the opposite must also be excluded, where virtual classrooms are assigned to classes that are face-to-face. This condition is expressed in restriction (21).

$$(1 - Y_{dc,mt,'P'}) - W_{dc,mt,d,mh,'V'} \geq 0 \quad \forall (dc,mt) \in DM, (d,mh) \in DMH \tag{21}$$

Periodicity and Consecutivity Constraints

A major challenge in automated class scheduling lies in representing the duration of classes and their periodicity and/or consecutiveness conditions.

While these terms are commonly found in the reviewed works, their meaning may slightly vary from one author to another. In what follows, we will refer to the consecutiveness condition as the case where a certain number of time modules must be taught consecutively for a specific subject, and for the same group of students and instructors, but where the number of consecutive modules is not pre-established but is a decision variable. Furthermore, we will refer to the repeatability condition as the general case where a certain subject requires being taught on two or more days of the week to meet its programmed workload. The developed model simultaneously addresses both types of conditions through constraints (22), (23), (24), and (25).

Given constraint (22), for a determined day (*d*) and for a certain subject (*dc,mt*), there can be a unique starting module (*mh*) with its respective time duration (*h*).

$$\sum_{mh,h} H_{dc,mt,d,mh,h} \leq 1 \quad \forall d, (dc,mt) \in DM \tag{22}$$

On the other hand, restriction (23) forces the number of modules for the same subject on a given day to be equal to the selected number of hours (*h*). This ensures that if 0 hours are selected for the duration, no classes can be scheduled.

$$\sum_{mh,a} W_{dc,mt,d,mh,a} = \sum_{mh,h} h \cdot H_{dc,mt,d,mh,h} \quad \forall d, (dc,mt) \in DM \tag{23}$$

Constraints (24) and (25) prevent a class with duration *N* from starting when an insufficient number of time slots remain to complete it before the last time slot of the corresponding day.

$$\sum_{mh,a} W_{dc,mt,d,mh,a} \geq h \cdot H_{dc,mt,d,mh',h} \quad (Consecutividad) \tag{24}$$

$$\sum_{h\ldots} H_{dc,mt,d,mh,h} = 0 \quad \forall (dc,mt) \in DM, (d,mh) \ldots \tag{25}$$

Logic Constraints

In addition to the previously described constraints, there is a set of logical rules that link the binary decision variables *Y*, *UP*, and *W*.

Rule 1

If and only if it is selected that a subject *mt* will be taught by the teacher *dc* in a *face-to-face* format, then a day *d* and a classroom *a* will be assigned for said face-to-face class (variable *UP*). This is expressed by the logical constraint (26).

$$Y_{dc,mt,\mathbf{P}} \Leftrightarrow \bigvee_{d,a \in F} [UP_{mt,dc,d,a}] \quad \forall (dc,mt) \in DM \tag{26}$$

Rule 2

If it is selected that the teacher *dc* will teach a class of the subject *mt* on day *d*, during the time slot *mh* and in the physical classroom *a*, then the class must be *In-person*. This is expressed by the logical constraint (27).

$$W_{dc,mt,d,mh,a} \Longrightarrow UP_{mt,dc,d,a} \quad \forall (dc,mt) \in DM, d, a \in F \tag{27}$$

Rule 3

If it is selected that teacher *dc* imparts a class of subject *mt* on day *d* and room *a* in *in-person* format, then on said day (*d*) the class (*dc,mt*) must be avoided in rooms other than *a* for all time slots *mh* (variable *W*). This is expressed by the logical constraint (28).

$$UP_{dc,mt,d,a} \Longrightarrow \neg W_{dc,mt,d,mh,a'} \quad \forall (dc,mt) \in DM, d, a' \neq a \tag{28}$$

Implementation and Case Study

The model was implemented using the Pyomo framework (Hart et al., 2011) in its 6.2 version. The logical constraints were transformed into algebraic constraints using the rules described by Raman & Grossmann (1991), yielding a MILP model. The commercial solver IBM CPLEX was used to solve this model. The study was conducted on a computer with an AMD Ryzen 5 5600X 6-Core processor and 16 GB of RAM.

To evaluate the performance of the developed model, the planning process of a faculty with four university degree programs was taken as a case study. This planning requires coordinating the classes of 80 distinct courses, taught by 71 lecturers, in 18 classrooms of different capacities, and over 14 time slots for each of the 5 days of the weekly calendar. Some courses are shared among student groups from different degree programs and/or cohorts. Three of the degree programs have a duration of 5 years, while the fourth is 3 years long.

Solution, Results and Discussion

Two solution approaches were tested. In the first instance, the objective was to solve the original problem through the direct use of a commercial MIP solver (IBM-CPLEX). In the second approach, a heuristic method was implemented where the solution process is decomposed into two stages: (a) scheduling of classes, (b) planning of classroom spaces. For the case study, the model consists of 801 thousand constraints and 473 thousand variables (including binary and continuous).

For the optimization run of approach 1 (original problem), a maximum search time of 11 hours was configured. Meanwhile, for the run of approach 2 (schedule-classroom decomposition), a maximum of 1 hour was configured for the scheduling assignment stage and 10 hours for the class assignment.

The computational performance of both solution approaches is summarized in table 1 and illustrated in figure 1. As shown in figure 1 (A), the run time is drastically reduced from 39,601 seconds in the original formulation to only 155 seconds when using the decomposition method. This represents an acceleration of over 250 times, with the decomposition approach requiring less than 0.4% of the time used to solve the original problem. It is worth noting that the search for the original problem was terminated due to the pre-established time limit, which highlights the complexity of the non-decomposed model.

Table 1

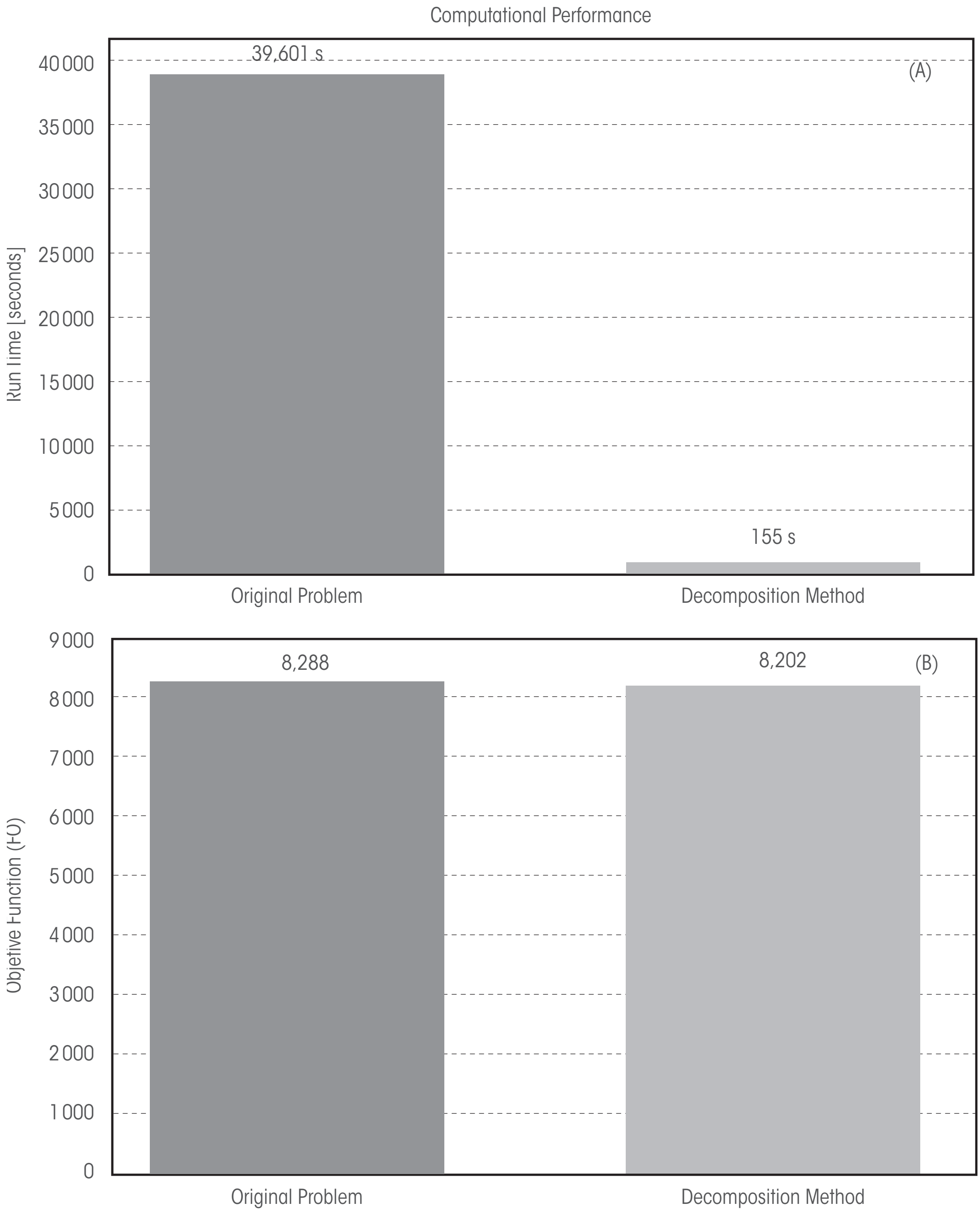
Performance of the solution process for the original problem and the decomposition method

Performance Parameter	Original Problem	Decomposition Method
Run time (seconds)	39,601.00	155.00
Objective function (FO)	8,288.00	8,202.00
FO up bound	8,727.00	--
In-person students	2,282.00	2,212.00

Despite this significant reduction in computational effort, figure 1 (B) demonstrates that solution quality is not compromised. The decomposition method yields an objective function value that is 1% lower (better) than the one achieved by the original formulation. Although the original model schedules approximately 3% more face-to-face students (2,282 vs. 2,212), the trade-off is negligible when considering the immense gain in computational tractability. These results confirm that the proposed hybrid algorithm provides a robust and high-quality solution within practical timeframes for institutional academic planning.

Figure 1

Computational performance



Note. Comparison of computational performance between the original problem formulation and the proposed decomposition method: (A) run time in seconds and (B) objective function (FO) values.

As observed in Table 1, the solution found after 11 hours of execution in the original formulation by a commercial solver does not represent a global optimum, given that a gap still exists between the feasible solution and the upper bound. However, in current institutional practice, planning processes often lead to solutions that, while potentially sub-optimal, must strictly satisfy all feasibility requirements. Future research should involve the application of this model in real-world scenarios to evaluate whether the solutions obtained are fully comparable in quality to those produced by expert human planners.

On the other hand, the application of decomposition heuristics has notably reduced the time required to obtain solutions while maintaining practically the same FO level. This drastically improved efficiency enables the possibility of performing automated re-factorizations or rescheduling of a plan in a matter of minutes. This stands in stark contrast to the original 10 or more hours execution times, which typically forced administrative staff to run calculations exclusively as overnight processes. This result is highly significant, as it could expedite the implementation of such systems as primary tools for course scheduling—a process that currently demands between one and two months of manual labor when performed without these computational aids.

Conclusions

In this work, a MILP model for optimal class scheduling based on the curriculum planning approach has been developed. This approach is particularly well-suited for the Argentine university system, as it manages the student body in an aggregate manner, making it significantly less data-intensive and enhancing computational tractability compared to individual-based models. A key contribution of this research is the successful integration of teaching modality—specifically the simultaneous planning of face-to-face and synchronous virtual classes—within a curriculum-based framework.

While the original model can be solved directly using commercial MILP solvers, the high computational demand limits its practical application in real-world settings. To overcome this, a hybrid decomposition method was implemented, achieving an efficiency gain of over two orders of magnitude (at least a 100x speedup). This approach allows for obtaining high-quality solutions in the order of a few minutes for an entire faculty, representing a drastic improvement over traditional 10-hour execution times.

In conclusion, the drastic reduction in processing time facilitates the rapid adoption of this model as a decision-support tool within current administrative workflows, potentially transforming a process that traditionally takes months into one that can be managed in minutes. Although further field testing in real-use environments is required to fully validate its institutional impact, the results demonstrate that the proposed decomposition algorithm is the most viable option for frequent re-scheduling and practical university management.

Future Research Directions

Although promising results have been found regarding the savings in computational cost of decomposition methods, it is necessary to evaluate how this cost grows as a function of the instance size. In turn, research should be conducted on the possibility that, in certain problem instances, the decomposition method may not find feasible solutions, even though they exist for the original problem, and on what to do to build more robust decomposition methods. Additionally, an alternative line of research involves the reformulation of the original model by replacing Big-M relaxations with Convex Hull formulations. This could theoretically accelerate the resolution of the original problem without sacrificing optimality, providing a tighter linear relaxation that enhances solver performance.

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